Numerical Integration

Compare the trapezoidal rule to the left Riemann sum. The area of each trapezoid is calculated using twice as much information (left *and* right endpoints) as the area of each rectangle. This leads one to expect that applying the trapezoidal rule with $n = 6$ should produce a result comparable to the one obtained from a Riemann sum with $n = 12$.

a) Open the Riemann Sums mathlet. Set the function to $x^3 - 2x$ and select the trapezoidal rule. Make sure $n = 6$. Record the mathlet's estimate of the integral.

b) Select "Evaluation point" to change to the Riemann sum approximation. Move the slider to 0.5, setting the evaluation point to be the midpoint of the interval. Set n equal to 12 and record the mathlet's estimate of the integral.

c) Calculate $\displaystyle\int_{-1}^{2} x^3 - 2x\,dx$ by hand. Was the accuracy of the Riemann sum with $n = 12$ comparable to that of the trapezoidal rule with $n = 6$? Why or why not?

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a) 0.93750

b) 0.72656

c) $\int_{-1}^{2} x^3 - 2x \, dx$

$= \left. \frac{x^4}{4} - \frac{2x^2}{2} \right|_{-1}^{2}$

$= \left(\frac{16}{4} - 4\right) - \left(\frac{1}{4} - 1\right)$

$= 0.750$

No, Riemann sum was more accurate.
This is because it used many more rectangles which leads to a better estimate of area.